Ron was Wrong, Whit is Right

Sanity checks on the Internet PKI

Arjen K. Lenstra [1] James P. Hughes [2]
Maxime Augier [1] Joppe Bos [1] Thorsten Kleinjung [1]
Christophe Wachter [1]

[1] Ecole Polytechnique Federale de Lausanne, LACAL, Station 14, CH-1015 Lausanne

[2] Self, Palo Alto, CA, USA

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Project goal

- Public key cryptography is widely used and subject to much scrutiny.
- But what about the implementations, and operational conditions ?
- Public keys are public. Let's collect as many as possible and perform some sanity checks.

Key sources

- PGP Keyservers: 5.4 M keys
- EFF SSL Observatory: 6.2M, then 7.2M keys (two datasets)
- Various other X509 collection projects
- Missing: SSH

Key statistics

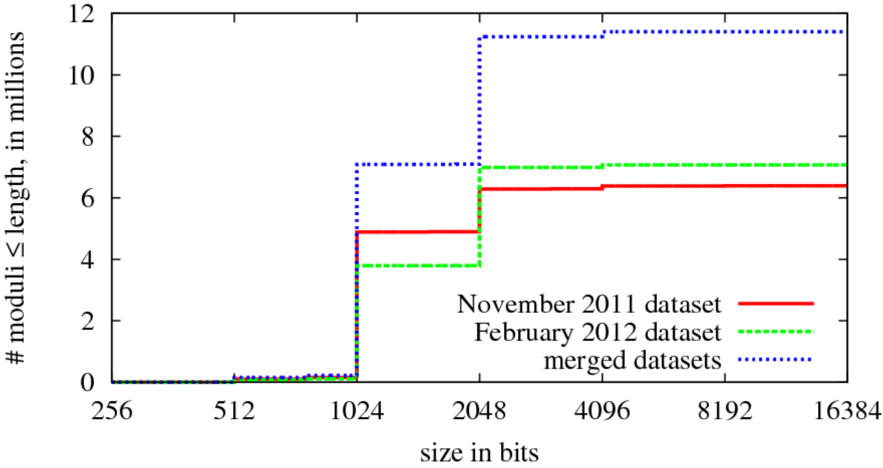
- Total *individual* PGP keys and subkeys collected: 5'481'332

ElGamal	2'546'752
DSA	2'536'959
RSA	397'621

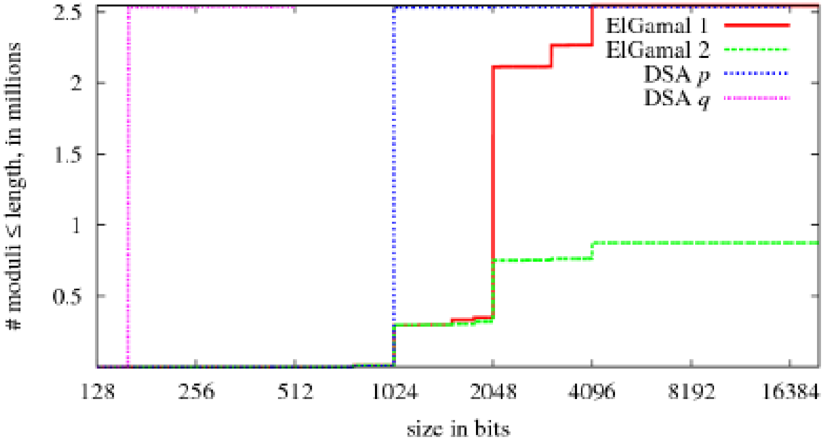
- Total distinct x509 keys: 6'185'372

RSA	6M
DSA	141
ECDSA	1

RSA Moduli sizes



ElGamal/DSA sizes



X509 Hash algorithms

SHA1 *	4.8M
MD5	1.35M
SHA256 *	5.3k
SHA512 *	525
MD2	122
GOST	30
SHA384 *	24
MD4	14
RIPEMD160	9

- 47.6%: Expiration date later than 2011
- 33.4%: above + SHA1 or better

1. Key Collection
2. RSA
   - Exponents
   - Moduli
   - Mutually Factorable keys
3. ElGamal, DSA, ECDSA
   - ElGamal
   - DSA
   - ECDSA
4. Conclusion

RSA reminder

- Pick p,q primes
- Compute $n = pq$
- Compute $\phi(n) = (p-1)(q-1)$
- Pick e coprime with $\phi(n)$
- Compute d $|ed \equiv 1 \pmod{\phi(n)}$
- $c \equiv m^e \pmod{n}, m' \equiv c^d \pmod{n}$
- $m' \equiv m^{ed} \equiv m^1 \pmod{n}$

What would obviously be wrong ?

- $e = 1$: ROT26
- e even: extremely hard to decrypt
- Suspiciously large/random e

What would obviously be wrong ?

- $e = 1$: ROT26 (8 occurrences)
- e even: extremely hard to decrypt (2 occurrences)
- Suspiciously large/random e (2 occurrences)

How did this happen ?

```
int_to_bignum(int e) {
        BIGNUM *big = BN_new();
        for (i=0; i < sizeof(exp); ++i)
                if (exp & (1 << i))
                        BN_set_bit(big, i);
```

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- Small e are safe if m is properly padded
- Special small e means fast encryption/verification
- e and d are interchangeable in key generation
- Hey, let's pick a special d to make decryption/signing fast !

Exponents distribution

X.509		PGP	
e	%	e	%
65537	98.49	65537	48.85
17	0.76	17	39.5
3	0.38	41	7.57
35	0.14	19	2.48
5	0.12	257	0.39
other	0.1	other	0.6

Duplicate moduli

We look for identical moduli in different certificates

- 4.3% (240k) of the X.509 certificates have non-unique moduli
- Among these, 30k (0.5%) Debian moduli
- Legitimate cases: same owner with different expiration dates
- Sometimes no obvious relation between them

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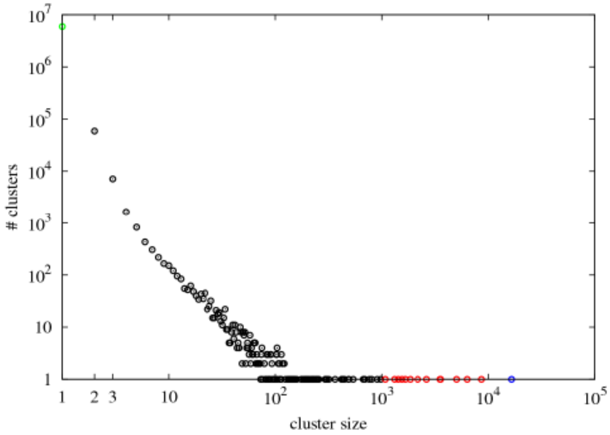
And in PGP keys

- 59 non-unique moduli

Moduli clusters



Broken moduli

Simple checks on the moduli:

- Prime n: $\phi(n) = n - 1$
- Small factors in n
  - Even n
  - Copy-paste mistakes
- Fermat method

Broken moduli

Simple checks on the moduli:

- Prime n: $\phi(n) = n - 1$ (2 occurrences)
- Small factors in n (171 occurrences)
  - Even n (68 occurrences)
  - Copy-paste mistakes (9 occurrences)
- Fermat method (0 occurrences)

Actual fun stuff

- User 1 picks $n_1 = p_1 q_1$
- User 2 picks $n_2 = p_2 q_2$

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- User 1 picks $n_1 = p_1 q_1$
- User 2 picks $n_2 = p_2 q_2$
- What if $p_1 = p_2 = p$?
- Anyone can compute $gcd(n_1, n_2) = p$, and recover p, q_1, q_2.
- Thus, these moduli offer no security.

Checking for colliding keys

- GCD is fast ($O(N \log^2 N)$).
- However, colliding keys requires to check pairs. Naively doing so ($O(n^2)$) is unworkable.
- However, well-known solutions exist. We used a LCM-tree while checking each node for a non-trivial GCD, which runs close to $O(n \log n)$.

Collision shapes

Consider the undirected graph where vertices are prime numbers, and edges are existing keys.

- Ideally, this would be a forest of disconnected edges.
- In practice: 1995 components of more than 1 edge (1st run).
- 1988 are trees of depth one (single magic prime per tree)
- Total 27k/31k total (0.4%) keys compromised

How did this happen ?

```
seed_my_rng();
PRIME p = random_prime();
PRIME q = random_prime();
```

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```
seed_my_rng();
PRIME p = random_prime();
PRIME q = random_prime();

void seed_my_rng() {
    state = 4; //chosen by fair dice roll
}
```

How did this happen ?

```
seed_my_rng();
PRIME p = random_prime();
//just to be safe
seed_my_rng_again()
PRIME q = random_prime();
```

How did this happen ?

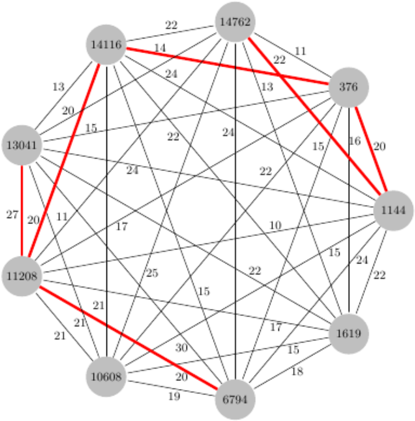
```
PRIME random_prime() {
    PRIME p;
    do {
        p = random_number();
    } while (!prime(p)); // I Feel Lucky !
    return p;
}
```

Suspect 1: Network device manufacturer X

- Bunch of X509 self-signed certificates
- Suspiciously similar generation date, Jan 01 20XX 00:00:0Y
- A single magic p in common for several thousand certs
- Duplicate keys on seemingly unrelated devices. *The moduli are correlated with the generation times*

Suspect 2: Embedded management device Y

- Fully connected set of *9* distinct primes
- Each modulus occurs in many keys with unrelated owners.



Disclosure

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"Why not set up an online service to check weak keys ?"

- A key may be flawed and we won't have any idea until another key comes along.
- Telling that a key is weak implicitely compromises another one.
- Our data sources are public. Allowing anyone to test the keys would let an attacker filter out the weak keys and massively speed up the attack.

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ElGamal reminder

- Pick a prime p
- Pick $g \in (\mathbb{Z}/p\mathbb{Z})^*$
- Pick x
- Compute $y \equiv g^x \pmod{p}$

Basic checks

Total keys: 2.55M

- p not prime: 82 occurrences
- p not safe prime: 34.4%
- g does not generate $(\mathbb{Z}/p\mathbb{Z})^*$: at least 16.4%
- $y \notin \langle g \rangle$: 33 occurrences
- a few suspicious y values

DSA reminder

- Pick primes p,q with $q|(p-1)$
- Pick g such that $\langle g \rangle$ is of order q.
- Pick secret key x $0 \leq x < q$
- Compute $y \equiv g^y \pmod{p}$

Basic tests

- p not prime
- q not prime
- q does not divide $p-1$
- g not of order q
- y not of order q
- $x < 2^{12}$
- Copy-paste errors again ?

Basic tests

- p not prime (12 occurrences)
- q not prime (2 occurrences)
- q does not divide $p-1$ (10 occurrences)
- g not of order q (no occurrences)
- y not of order q (42 occurrences)
- $x < 2^{12}$ (no occurrences)
- Copy-paste errors again ?

ECDSA

- Only one key (no ssh keys yet)
- Only a dozen of signatures

Conclusions

- Most of the observed PKI appears to be working as intended
- 20k RSA moduli have been factored in 30k certs (0.5% of the collection)
- Requiring multiple secrets may have non-obvious pitfalls (Ron vs Whit thing)
- Can we trust nonce randomness where it is critical ?

Conclusions

- Most of the observed PKI appears to be working as intended
- 20k RSA moduli have been factored in 30k certs (0.5% of the collection)
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and NOT (as it has been misreported):

- That this makes RSA itself broken or inferior
- That electronic commerce will collapse